Eastern Hills Senior High School

MATHEMATICS METHODS UNITS 1 AND 2
Section One:
Calculator-free

SOLUTIONS

Student Number: In figures

In words   _____

Your name   _____

Time allowed for this section
Reading time before commencing work: five minutes
Working time for section: fifty minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	7	7	50	52	35
Section Two: Calculator-assumed	12	12	100	98	65
			Total	150	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer Booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

5. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

Section One: Calculator-free **(52 Marks)**

This section has **seven (7)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 50 minutes.

Question 1 **(5 marks)**

Calculate the value of

(a) $16^{-0.5}$. (2 marks)

$$16^{-0.5} = \frac{1}{\sqrt{16}}$$
$$= \frac{1}{4}$$

(b) $\left(a \div b\right)^2$ when $a = 4 \times 10^2$ and $b = 8 \times 10^3$, leaving your answer in scientific notation.

(3 marks)

$$\left(\frac{4 \times 10^2}{8 \times 10^3}\right)^2 = \left(0.5 \times 10^{-1}\right)^2$$
$$= 0.25 \times 10^{-2}$$
$$= 2.5 \times 10^{-3}$$

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Question 2 **(6 marks)**

A line has equation $y = mx + 3$. Determine the value of the constant m if the line

(a)   is parallel to $4x + 2y = 1$.   (1 mark)

$$y = -2x + \tfrac{1}{2} \implies m = -2$$

(b)   is perpendicular to $y = -2x + 3$.   (1 mark)

$$-2 \times m = -1 \implies m = \frac{1}{2}$$

(c)   passes through the point of intersection of the lines $y = \dfrac{2x + 10}{3}$ and $y = 6 - 2x$.   (4 marks)

$$\frac{2x + 10}{3} = 6 - 2x$$
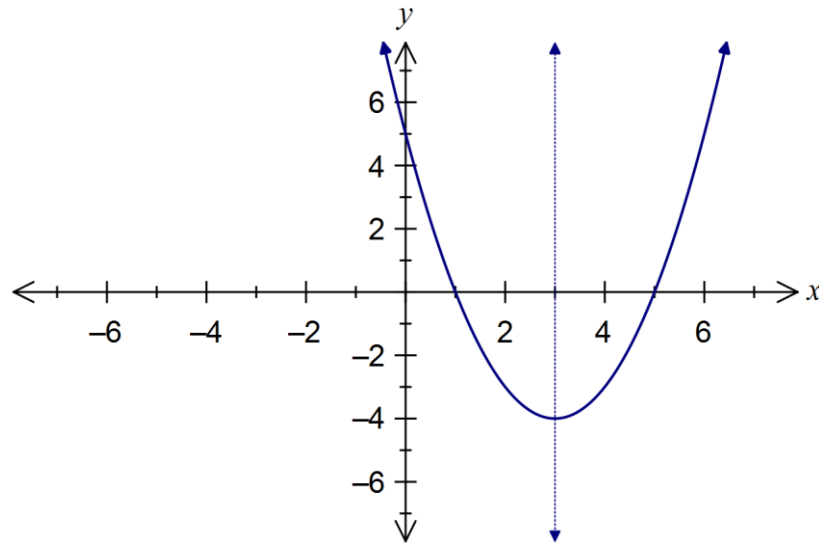
$$2x + 10 = 18 - 6x$$
$$8x = 8$$
$$x = 1$$

$$y = 4$$

$$4 = m + 3 \implies m = 1$$

See next page

Question 3 **(8 marks)**

(a) A parabola passes through the points (0, 5), (5, 0) and has $x = 3$ as its axis of symmetry.

    (i) Sketch the graph of this parabola. (3 marks)

    (ii) Determine the equation of the curve in the form $y = ax^2 + bx + c$. (2 marks)

$$y = a(x - 1)(x - 5)$$
$$5 = a(0 - 1)(0 - 5) \implies a = 1$$

$$y = x^2 - 6x + 5$$

(b) Determine the discriminant of the quadratic expression $2x^2 - 8x + 9$ and hence state the number of zeroes of the expression. (3 marks)

$$a = 2, b = -8, c = 9$$

$$b^2 - 4ac = 64 - 72 = -8$$

Since $b^2 - 4ac < 0$, there are no zeroes.
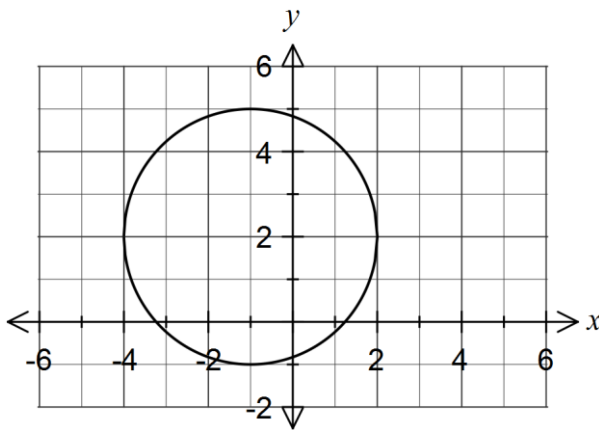
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Question 4 **(8 marks)**

(a) State the coordinates of all axes intercepts of the graph of $x^2 + y^2 = 9$. (2 marks)

(3, 0), (-3, 0), (0, 3) and (0, -3).

(b) State the equation of the axis of symmetry of the graph of $x = y^2$. (1 mark)

$y = 0$

(c) The graph of a relationship is shown below.

(i) Determine the equation of the graph. (2 marks)

$(x+1)^2 + (y-2)^2 = 3^2$

(ii) Calculate the exact coordinates of the positive x-axis intercept. (3 marks)

$$y = 0$$
$$(x+1)^2 = 5$$

$$x + 1 = \pm\sqrt{5}$$

$x = \sqrt{5} - 1$ (+ve intercept only)
$(\sqrt{5} - 1, 0)$

See next page

Question 5 **(9 marks)**

(a) Determine $\dfrac{dy}{dx}$ for

(i) $y = \dfrac{4x^4}{3}$. (1 mark)

$$\dfrac{dy}{dx} = \dfrac{16x^3}{3}$$

(ii) $y = \dfrac{12}{\sqrt{x}}$. (2 marks)

$$y = 12x^{-0.5}$$

$$\dfrac{dy}{dx} = -6x^{-1.5}$$

(b) Determine $f'(2)$ if $f(x) = \dfrac{x^2}{4} - \dfrac{4}{x}$. (3 marks)

$$f'(x) = \dfrac{x}{2} + \dfrac{4}{x^2}$$
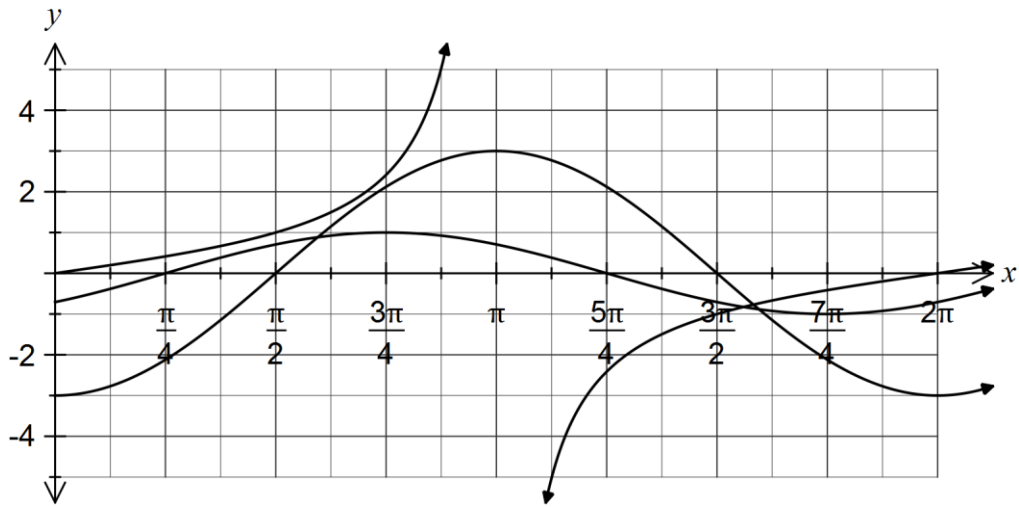
$$f'(2) = 1 + 1$$
$$= 2$$

(c) Determine $g(x)$ if $g(1) = -1$ and $g'(x) = 2x^2 + \dfrac{2x}{3} + 5$. (3 marks)

$$g(x) = \dfrac{2x^3}{3} + \dfrac{x^2}{3} + 5x + c$$

$$-1 = \dfrac{2}{3} + \dfrac{1}{3} + 5 + c$$

$$c = -7$$

$$g(x) = \dfrac{2x^3}{3} + \dfrac{x^2}{3} + 5x - 7$$

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Question 6   **(6 marks)**

(a)   The graphs of $y = \tan(ax)$, $y = b\cos(x)$ and $y = \sin(x + c)$ are shown below.

Determine the values of the constants a, b and c.   (3 marks)

$$a = \frac{1}{2} \qquad b = -3 \qquad c = -\frac{\pi}{4}$$

(b)   Solve the equation $\sqrt{3}\cos\left(x - \dfrac{\pi}{2}\right) = \cos(x)$ for $0 \leq x \leq 2\pi$.   (3 marks)

$$\sqrt{3}\cos\left(x - \frac{\pi}{2}\right) = \cos(x)$$

$$\sqrt{3}\sin(x) = \cos(x)$$

$$\frac{\sin(x)}{\cos(x)} = \tan(x) = \frac{1}{\sqrt{3}}$$

$$x = \frac{\pi}{6}, \frac{7\pi}{6}$$

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Question 7 **(10 marks)**

(a) Expand $(x-2)^4$. (3 marks)

$$(x-2)^4 = x^4 + 4x^3(-2) + 6 \times x^2(-2)^2 + 4 \times x(-2)^3 + (-2)^4$$

$$= x^4 - 8x^3 + 24x^2 - 32x + 16$$

(b) Solve the following for x:

(i) $4^{2x-1} = \dfrac{1}{8}$. (3 marks)

$$LHS = 2^{2(2x-1)}$$

$$RHS = 2^{-3}$$

$$4x - 2 = -3$$

$$x = -\dfrac{1}{4}$$

(ii) $x^3 - x^2 - 17x - 15 = 0$. (4 marks)

$$x = -1, \quad -1 - 1 + 17 - 15 = 0$$

$$x^3 - x^2 - 17x - 15 = (x+1)(x^2 - 2x - 15)$$

$$= (x+1)(x+3)(x-5)$$

$$x = -1, \ -3, \ 5$$

End of questions

Additional working space

Question number: _____

Additional working space

Question number: _____